

**MXB226**

**Case Study Project**

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**1. Executive Summary**

# This report modelled and solved the steady-state heat distribution in a custom electronic component.

# The electronic component has a sensitive point at (0.03, 0.03). This point will remain within the functionable 50°C to 55°C range if the ambient temperature the component is exposed to remains within a XX°C and XX°C range.

# Five different storage methods were used when generating and solving the mathematical model. The most efficient storage method is X….

# 2. Introduction

## 2.1 Purpose of the report

The purpose of this report is to investigate the steady-state heat distribution in a newly designed custom electronic component and provide its performance specifications. A mathematical model for the heat distribution in the component has been developed and solved using numerical strategies. This report also investigates the efficiencies of the various mathematical solving and data storage methods.

The schematic of the electronic component is shown in Figure 1. The placement of the component within the electronic device results in different temperatures along the boundaries of the component. The boundary AB (red) maintains perfect thermal contact with another component, which has a known temperature of 70°C. The boundary CD (purple) also maintains perfect thermal contact with another component, which has a known temperature of 40°C. The boundary AED (green) is thermally insulated, and the boundary BC (blue) is exposed to the ambient air temperature.

Chart, line chart

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*Figure X: Schematic of electronic component.*

## 2.2 Issues to be discussed and their significance

The custom electronic component is to be marketed globally, so the component needs to meet specific performance specifications for this to occur. The component must maintain a temperature between 50°C and 55°C at the point (0.03, 0.03). This point (orange) is shown on Figure X. The component will not function properly outside of these temperatures. This report will investigate the ambient temperatures that the electronic component can be exposed to and maintain a workable temperature at that point.

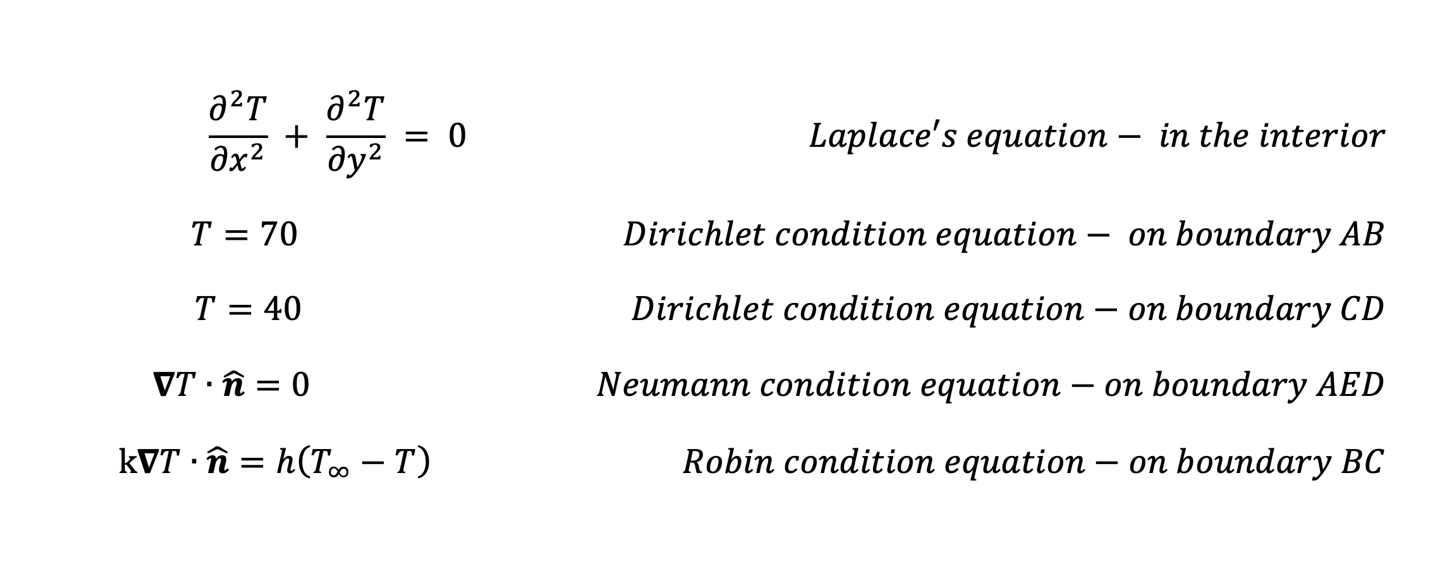
## 2.3 Limitations and assumptions

- model only – all models have inherent assumptions, can not perfectly represent reality

# 3. Discussion

## 3.1 Mathematical Model

The mathematical model for steady-state heat distribution, in the absence of sources or sinks, is given by Laplace’s equation, . Let *)* represent the temperature of the electronic component at the point *()*. The mathematical model for the component is as follows:



### The thermal conductivity, *k* = 3 *Wm-1C-1*, and heat transfer coefficient, *h* = 20 *Wm-1C-1* are known and remain constant throughout the investigation. Initially, the ambient temperature assumed as *T*∞ = 20°*C*.

### The electrical component model is converted to a matrix problem by discretising the surface area domain and forming finite difference equations. The domain is discredited by converting the continuous surface area into discrete counterparts using a mesh of squares. A node is assigned to each corner of the mesh, and numbered as shown in Figure X. The nodes along boundaries AB and CD are under prescribed temperature (Dirichlet) boundary conditions, meaning the temperature at those nodes will remain constant, therefore those nodes are not assigned a number. All nodes along the AB boundary remain a constant 70°*C* and all nodes along the CB boundary remain a constant 40°*C*.

### 

*Figure X: Discretisation mesh with assigned nodes.*

### 3.1.1 Laplace’s equation in discretised form

### To solve the mathematical model of heat distribution, Laplace’s equation needs to be converted to a discretised form. The second order partial derivatives in Laplace’s equation are approximated using a second order central difference approximation, derived from the Taylor series expansion, as follows:

### Where the nodes are assigned as shown in Figure X.

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*Figure X: Explanation of node index notation*

### Substituting (1) and (2) into Laplace’s equation:

### In this case , since the mesh is square, the equation can be arranged to:

### Equation (3) is the discretised form of Laplace’s equation and is used when solving for each node.

### 3.1.2 Solving for nodes

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*Figure X: Schematic defining interior nodes (blue) and unit normals for boundary nodes.*

### 3.1.2.1 Solving for Node 1

### Node 1 is under insulated boundary conditions, so is solved using the Neumann condition equation. This equation requires a unit normal, as shown in green in Figure X.

### Let node 1 = u1, node 2 = u2, node 7 = u7, and let uW be the ghost node west of node 1 and uS be the ghost node south of node 1, as shown in Figure X.

### 

*Figure X: Insulated boundary node 1 showing ghost nodes in grey*

### Applying Equation (3) to this node gives:

### Solve for uW and uS using the insulated (Neumann condition) boundary equation, .

### 

*Figure X: Unit normal for Node 1*

### Substituting (5) into (5).

### 3.1.2.2 Solving for ED boundary nodes

### The nodes along the ED boundary (2, 3, 4, 5 and 6) are under insulated boundary conditions, so are solved using the Neumann condition equation. This equation requires a unit normal, as shown in orange in Figure X.

### Using Node 3 as an example, let node 3 = u3, node 2 = u2, node 4 = u4, node 9 = u9, and let uS be the ghost node south of node 3, as shown in Figure X.

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*Figure X: Example of insulated ED boundary node showing ghost node in grey*

### Applying Equation (3) to this node gives:

### Solve for uS using the insulated (Neumann condition) boundary equation, .

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*Figure X: Unit normal for ED boundary node*

### Substituting (7) into (6).

### 3.1.2.3 Solving for AE boundary nodes

### The nodes along the AE boundary (7, 13, 19, 25 and 30) are under insulated boundary conditions, so are solved using the Neumann condition equation. This equation requires a unit normal, as shown in purple in Figure X.

### Using Node 13 as an example, let node 13 = u13, node 7 = u7, node 14 = u14 and node 19 = u19, and let uW be the ghost node west of node 13, as shown in Figure X.

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*Figure X: Example of insulated AE boundary node showing ghost node in grey*

### Applying Equation (3) to this node gives:

### Solve for uW using the insulated (Neumann condition) boundary equation, .

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*Figure X: Unit normal for AE boundary node*

### Sub (9) into (8).

### 3.1.2.3 Solving for Node 29

### Node 29 is under convective boundary conditions, so is solved using the Robin condition equation. This equation requires a unit normal, as shown in blue in Figure X.

### Let node 29 = u29, node 28 = u28, node 23 = u23, and let uE be the ghost node east of node 29 and uN be the ghost node north of node 29, as shown in Figure X.

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*Figure X: Convective boundary node 29 showing ghost nodes in grey*

### Applying Equation (3) to this node gives:

### Solve for uE and uN using the convective (Robin condition) boundary equation, .

### 

*Figure X: Unit normal for Node 29*

### Substituting (9) into (8) and substituting in constants:

### 3.1.2.4 Solving for Node 24

### Node 24 is under convective boundary conditions, so is solved using the Robin condition equation. This equation requires a unit normal, as shown in red in Figure X.

### Let node 24 = u24, node 23 = u23, node 18 = u18, and let uN be the ghost node north of node 33, as shown in Figure X. The node east of node 24 is known to be 40°C.

### Chart Description automatically generated

*Figure X: Convective boundary node 24 showing ghost node in grey*

### Applying Equation (3) to this node gives:

### Solve for uN using the convective (Robin condition) boundary equation, .

### Diagram Description automatically generated

*Figure X: Unit normal for Node 24*

### Substituting (11) into (10).

### Substituting in *a* and *b* and constants.

### 3.1.2.5 Solving for Node 33

### Node 33 is under convective boundary conditions, so is solved using the Robin condition equation. This equation requires a unit normal, as shown in yellow in Figure X.

### Let node 33 = u33, node 32 = u32, node 28 = u28, and let uE be the ghost node east of node 33, as shown in Figure X. The node north of node 33 is known to be 70°C.

### 

*Figure X: Convective boundary node 33 showing ghost node in grey*

### Applying Equation (3) to this node gives:

### Solve for uE using the convective (Robin condition) boundary equation, .

### Diagram Description automatically generated

*Figure X: Unit normal for Node 33*

### Substituting (12) into (11):

### Substituting in *a* and *b* and constants:

### 3.1.2.6 Solving for interior nodes

### The interior nodes, as shown in blue in Figure X, are not under any boundary conditions, so the interior nodes are solved using the standard Laplace’s equation.

### Using Node 15 as an example, let node 15 = u15, node 9 = u9, node 13 = u13, node 16 = u16, and node 21 = u21, as shown in Figure X.

### 

*Figure X: Example of interior node*

### Applying Equation (3) to this node gives:

### 2.1.2 Numerical Approach

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### 2.1.3 Direct Methods

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### 2.1.4 Iterative Methods

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### 2.1.3 Node Re-Orderings

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## 2.2 Discussion and analysis of data

### 2.2.1 Method Performance

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### 2.2.2 Issue 2

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### 2.2.3 Issue 3

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### 2.2.4 Reliability and accuracy of data

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# 3. Conclusions

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# 4. Recommendations

## 4.1 Recommendation 1 – component specifications?

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## 4.2 Recommendation 2 – data storage efficiency?

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# 5. References

Chapter Notes – how to reference

# 6. Appendices

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